Edward Frenkel

AMS Einstein Public Lecture: Imagination and Knowledge

It is an honor to give the AMS Einstein Public Lecture this year. The title of my lecture is "Imagination and Knowledge." As this is a public lecture, I would like to talk about the essence of our profession, its role in today's world, and the responsibilities that come with it. We live in the age of artificial intelligence, with math-driven information technology invading our lives at an accelerating pace, bringing us new opportunities and unprecedented connectivity but also multiple challenges to our sense of identity and reality, our culture, and even the truth. It's not surprising that in this environment many people look to mathematicians, expecting us to provide some clarity and perhaps even some guidance on how to navigate today's world. For example, a famous author recently asked me, "Is life just an algorithm?" In fact, that's what I often hear, even from some super-smart folks: life is an algorithm; love: just a chemical reaction; a human: nothing but a sequence of 0s and 1s. But is it really true? That's what I want to talk about in this lecture.

I want to start with a quote from Albert Einstein: "Imagination is more important than knowledge. For knowledge is limited, whereas imagination embraces the entire world." In my talk, I will give examples from the history of mathematics that support Einstein's view. These examples show how imagination provides bursts of insight that enable mathematicians to make new advances and to abandon what was taken for granted as well known and well understood.

My first example is the discovery of imaginary numbers (no pun intended!) in Gerolamo Cardano's wonderful treatise *Ars Magna* (circa 1545, Figure 1). At the time, people thought that the square root of a negative number could not possibly exist. How could it, if the square of any real number is always nonnegative? Everybody knew that, case closed. But Cardano did not take this "knowledge" for granted. Instead, he dared to imagine that such numbers existed and tried to use them (specifically, the square root of −15) to tackle mathematical problems he was interested in. And though in the process of doing it he endured, in his own words, "mental tortures", he did succeed, proving that the idea was viable. Thus, imaginary numbers were born. Today, we cannot imagine our life without them. Moreover, they are not just a clever trick or a tool. They are essential elements of quantum mechanics, describing the fundamental structure of physical reality.

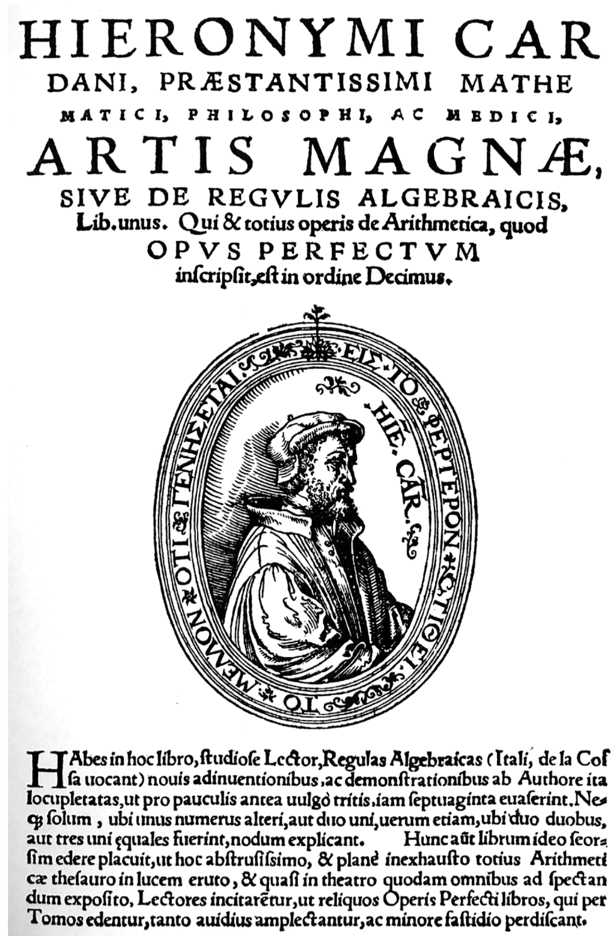

Figure 1. In his 1545 treatise *Ars Magna*, Gerolamo Cardano introduced imaginary numbers.

Without imaginary numbers, there is no Heisenberg uncertainty principle, no double slit experiment, no Bell's inequality. No reality as we know it.

As my second example, I want to talk about the great Indian mathematician Srinivasa Ramanujan (Figure 2), who was not formally trained in mathematics but made astounding discoveries that dazzle us to this day. He said that many of his formulas were shown to him in his dreams by the family deity, Goddess Namagiri. As soon as he woke up, he would commit them to writing, filling his famous notebooks (the last one was found by George Andrews in 1976 in a box of papers at the Trinity College of Cambridge University). Ramanujan didn't know how to prove most of his marvelous formulas. As G. H. Hardy, who became Ramanujan's mentor and patron, put it, "They must be true because, *if they were not true, no one would have had the imagination to invent them*" [1].

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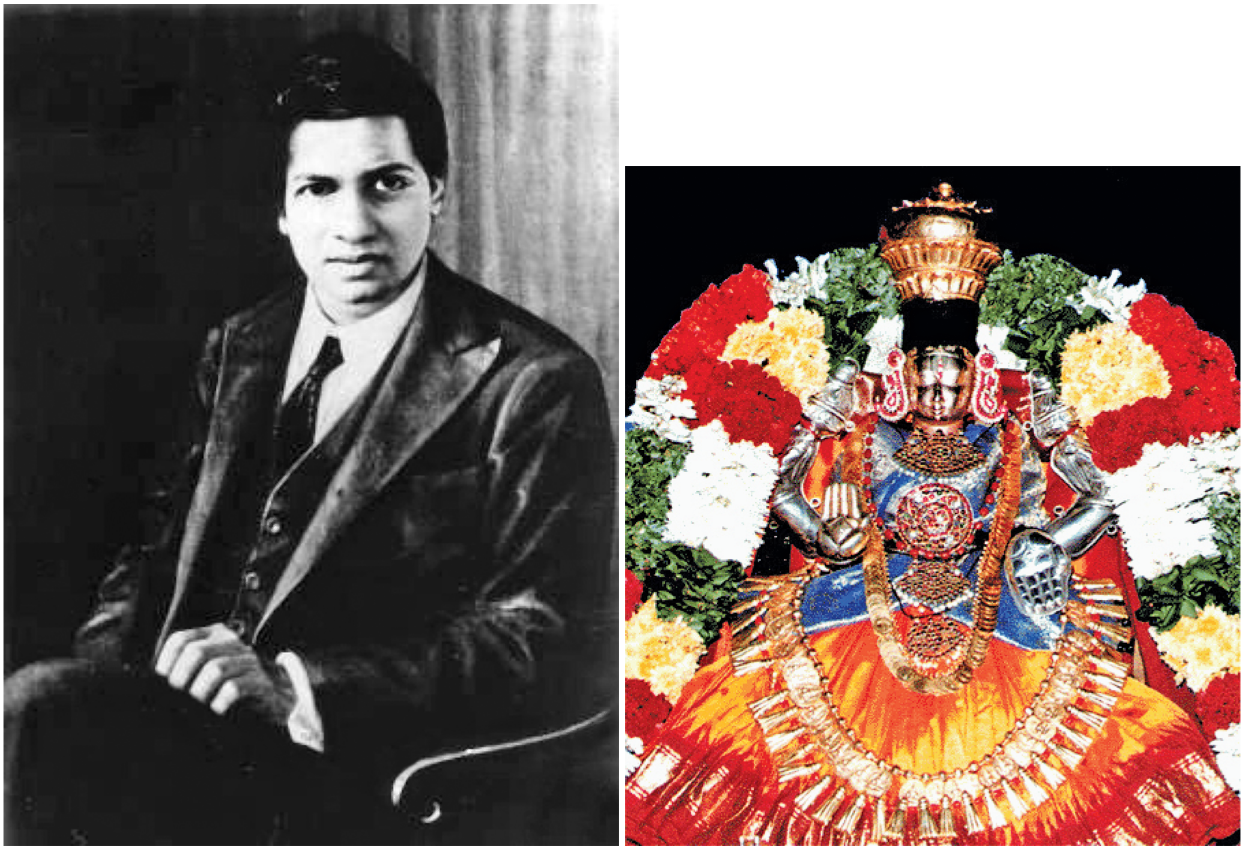

Figure 2. (a) The great Indian mathematician Srinivasa Ramanujan said that many of his formulas were shown to him in his dreams by the family deity, (b) Goddess Namagiri.

But it took several generations of mathematicians to supply the missing proofs.

"The human mind infinitely surpasses the powers of any finite machine."

—Kurt Gödel

Perhaps not all mathematicians are visited by a goddess in their dreams, but imagination certainly plays a crucial role in our work. My research is in the Langlands Program, an ambitious project launched by Robert Langlands in the late 1960s aimed at unifying different areas of mathematics. Langlands's original work connected number theory with harmonic analysis in novel and unexpected ways, and in later years similar patterns were found in geometry and even in quantum physics. Studying these ideas, it is quite clear that they were discontinuities in the path of linear knowledge; they could only be brought out by unbridled imagination.

When we look at these examples, a bigger point emerges: a mathematician's mind is not a computer. Indeed, our intuition and imagination cannot be accounted for by computation alone (though, of course, computation plays an essential role in our work). Or, as the great logician Kurt Gödel (Figure 3) convincingly argued in his 1951 Gibbs Lecture to the American Mathematical Society, "The human mind infinitely surpasses the powers of any finite machine." Gödel is echoed by Roger Penrose, who writes, "There is something in our conscious thought process that eludes computation.…We have access to mathematical truths that are beyond any robot's capabilities" [2]. It is essential for us mathematicians to appreciate this insight and to share it with others; it sheds light on questions such as, Is life an algorithm?

And that leads us to another lesson: just as mathematicians need to acknowledge, embrace, and utilize their capacity for imagination in order to be successful, so do all humans. Carl Jung [3] warned us that the widening rift between imagination and knowledge (or rather, "what we think we know") is a symptom of "the split consciousness so characteristic of the mental disorders of our day." This split has ushered in a brave new world in which multinational corporations are permitted to derive enormous

Figure 3. Albert Einstein with Kurt Gödel, who argued that "the human mind infinitely surpasses the powers of any finite machine."

profits from modifying and controlling humans' behavior, while we largely remain silent. In essence, it's the old adage of "divide and conquer"—exploiting our fears, which are always based on what we think we know about ourselves and others—now supercharged with AI-powered information technology. To counter that, we need to use our imagination, which has always been humanity's best antidote to dogma and oppression. Imagination is our way to survive and uphold our dignity. For it's the imagination that gives us a fresh start; it's what reminds us about our common goals and aspirations; it's what unites us. Dare to imagine.

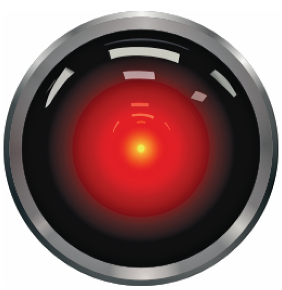

Figure 4. To counter the computer HAL 9000 of Stanley Kubrick's *2001: A Space Odyssey* and recent AI-powered information technology, we need to use our imagination. That's what gives birth to the "Star Child" in the film.

References

[1] G. H. Hardy, The Indian Mathematician Ramanujan, *Amer. Math. Monthly* **44**, (1937), no. 3, 137–155. MR1523880

[2] R. Penrose, *Shadows of the Mind*, Oxford University Press, 1996. MR1865778

[3] C. Jung, *The Undiscovered Self*, Routledge, 2013.

Image Credits

Figure 1 and parts of Srinivasa Ramanujan's (Figure 2) are in the public domain.

Part b of Figure 2: image of Namagiri Thayar from the Anudinam Sri Vaishnava New Portal. "Namakkal Sri Narasimhaswami Temple Navarathri Utsava Patrikai." Uploaded September 25, 2014, by author Mugunthan. Accessed December 29, 2017. `anudinam.org/2014/09/25/namakkal-sri-narasimhaswami-temple-navarathri-utsava-patrikai`.

Figure 3 courtesy of Oskar Morgenstern, photographer. From the Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, New Jersey.

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ABOUT THE AUTHOR

Edward Frenkel is a member of the American Academy of Arts and Sciences, Fellow of the American Mathematical Society, and the winner of the Hermann Weyl Prize in mathematical physics. He is the author of a *New York Times* bestseller *Love and Math*, which has been published in eighteen languages.